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Analytical Solution for Flow and Concentration Boundary Layer of an Unsteady Three – Dimensional Stretching Surface

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Abstract

The effect of unsteady three dimensional flows over a stretching surface is studied. The governing boundary layer equations are transformed to ordinary differential equations. These equations are solved analytically using the optimal modified Homotopy Asymptotic method in order to get a closed form solution for the dimensionless functions f,g and ι. The velocity and concentration profiles are plotted and discussed in details for various values of the different embedded flow parameters.

Keywords: Optimal homotopy asymptotic method, stretching surface, boundary layer concentration, three dimensional flows.

1. Introduction

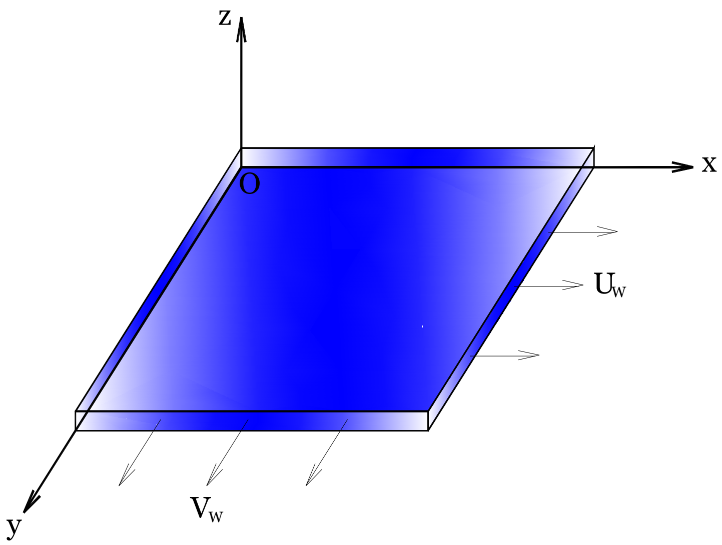
Many important engineering mass transfer processes occur simultaneously with heat transfer. Cooling towers, dryers, and combustors are just few examples of equipment that intimately couple heat and mass transfer. Coupling can arise when temperature-dependent mass transfer processes cause heat to be released or absorbed over a stretching surface. For example during evaporation latent heat is absorbed at a liquid surface when vapor is created. This tends to cool the surface, lowers the vapor pressure, and reduces the evaporation rate. Elbashbeshy et. al. [1-3] studied the effect of internal heat generation and suction/injection on the flow and thermal boundary layer over a stretching surface. Fang et. al. [4] studied the influence of temperature dependent viscosity and thermal conductivity on the boundary layers. Ishak et. al. [5-6] studied the effect of heat flux and suction/injection on the flow and thermal boundary layer over an unsteady stretching surface. Sultana et. al. [7] studied the effect of internal heat generation, suction/injection, and radiation on the flow over a stretching surface embedded in porous medium. Al-Odat et. al. [8] provided a local similarity solution of an exponentially stretching surface with an exponential dependence of the temperature distribution in the presence of the magnetic field effect. Rashed [9] studied the radiative effect on heat transfer from a non-isothermal, arbitrary stretching surface in a porous medium. EL-Arabawy [10] studied the effects of suction/injection on mass transfer over a stretching surface. All these studies considered the two dimensional flow problem; by developing the problem to three dimensional flows we found a good list of references which discussed this problem. Nazar et. al. [11] studied the effects of visco-elastic fluid on the velocity profiles of three dimensional flows over a stretching surface. Takhar et. al. [12] Studied the effects of heat transfer on three dimensional MHD boundary layer flow through a stretching surface. Kandasamy et. al. [13] studied the effects of variable viscosity, Heat and Mass transfer on nonlinear mixed convection flow over a porous wedge with heat radiation in the presence of homogenous chemical reaction. Shateyi [14] studied the Thermal Radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing. El-dabe et. al. [15] studied the effects of heat generation / absorption on three dimensional viscoelastic fluids through a stretching surface. Aboeldahab et. al. [16] Studied the combined free convective heat and mass transfer effects on the unsteady three dimensional laminar flow over a time dependent stretching surface, also the effect of generation or consumption of the diffusion species due to a homogeneous chemical reaction. Olanrewaju et. al. [17] Studied the effects of soret and dufour on an unsteady mixed convection past a porous plate moving through a binary misture of chemically reacting fluid. Elbashbeshy et. al.[19] studied the effects of the thermal radiation, heat generation and first order chemical reaction on heat and mass transfer on a steady three dimensional flow over stretching surface.

The purpose of this work is to study the effects of the thermal radiation, heat generation on heat and mass transfer on an unsteady three dimensional flow over stretching surface. It may be remarked that the present analysis is an extension of and a complement to the earlier paper [19].

2. Formulation of the Problem

Consider an unsteady, laminar, incompressible, and viscous flow on a continuous stretching surface. The concentration of species far from the surface, C_∞ is very small [18]. The x- axis, y- axis is run along the plan of a continuous surface, and the z-axis is perpendicular to it as shown in fig (1). The conservation equations for the steady three dimensional flows are

Figure 1: Physical model and coordinate system.



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\upsilon\frac{\partial^2 u}{\partial z^2} \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\upsilon\frac{\partial^2 v}{\partial z^2} \tag{3}$$

$$\frac{\partial c}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D\frac{\partial^2 C}{\partial z^2} \tag{4}$$

Where u, v and w are velocity components in the x, y, and z directions, respectively, υ is the viscosity ρ is the density of fluid, C is the concentration of the flow,and D is the effective diffusion coefficient.

It is assumed that the viscous dissipation is neglected, and the physical properties of the fluid are constant.

We assume that the stretching velocities Uw, and Vw, the and flow Concentration Cw are of the form

$$U_w = ax, \quad V_w = ay, \quad C_w = \frac{d_1 x^\alpha}{1-\gamma t} = \frac{d_2 y^\alpha}{1-\gamma t} \tag{6}$$

Where a is constant and called stretching rate, $d1$, $d2$, α and γ are constants.

We now introduce the following dimensionless functions f, g, and φ, and the similarity variable η

$$\eta = \sqrt{\frac{a}{\upsilon(1-\gamma t)}}z, \quad u = \frac{ax}{1-\gamma t} f'_{(\eta)}, \quad v = \frac{ay}{1-\gamma t} g'_{(\eta)}, w = -\sqrt{\frac{a\nu}{1-\gamma t}}(f+g), \quad \phi_{(\eta)} = \frac{C - C_\infty}{C_w - C_\infty} \tag{7}$$

Where prime denotes the differntiation with respect to η, using (7) the mass conservation equation (1) is indentically satisfied, and substituting into eqs. (2,3,and 4) we obtain

$$f''' + (f+g)f'' - f'^2 - \frac{\gamma}{a}(f' + \frac{\eta}{2} f'') = 0 \tag{8}$$

$$g''' + (f+g)g'' - g'^2 \frac{\gamma}{a}(g' + \frac{\eta}{2} g'') = 0 \tag{9}$$

$$\phi'' + Sc \ \ (f+g)\phi' - \alpha(f' + g')\phi - \frac{\gamma}{a}(\phi + \frac{\eta}{2}\phi' \ \ = 0 \tag{10}$$

($Sc = \upsilon/D$) is the schmidt number (11)

The boundary condition (6) become

$$f(0) = g(0) = \frac{1}{2}, \quad f'(0) = 1, \quad g'(0) = 1, \phi(0) = 1$$

$$f'(\infty) = 0, \quad f''(\infty) = 0, \quad g'(\infty) = 0, \quad g''(\infty) = 0, \quad \phi(\infty) = 0, \tag{12}$$

3. Optimal Homotopy Asymptotic Method (OHAM)

Consider a differential equation in the form:

$L(u(t)) + N(u)(t)) + g(t) = 0, \qquad B(u) = 0$ (13)

Where L is a linear operator, t denotes an independent variable, u(t) is an unknown function, g(t) is a known function, N(u(t)) is a nonlinear operator and B is a boundary operator. By means of OHAM a family of equations is constructed:

$(1-p)[L(F(t,p)) + g(t)] - H(p)[L(F(t,p)) + g(t) + N(F(t,p)) = 0, B(F(t,p)) = 0$ (14)

where $p \in [0,1]$ Is an embedding parameter, H(p) is a nonzero auxiliary function for $p \neq 0$ and H(0)=0, F(t,p) is an unknown function. Obviously, when p=0, and p=1, we have

$F(t,0)=u_0(t)$, $F(t,1)=u(t)$ (15)

Then, as p increases from 0 to 1, the solution F(t,p) varies from $u_0(t)$ to the solution u(t), where $u_0(t)$ is obtained from (14) for p=0:

$L(u_0(t))+g(t)=0, \qquad B(u_0) = 0$ (16)

The auxiliary function is chosen in the form;

$H(p)=pC1+p^2C2+$ (17)

Where C1, C2, … are constants which can be determined later.

Expanding F(t,p) in a series with respect to p, we get:

$$F(t,p,C_i) = u_0(t) + \sum_{k \geq 1} u_k(t, C_i) p^k$$

i=1,2, (18)

Substituting (18) in (14), collecting the same powers of p, and equating each coefficient of p to zero, we obtain a set of differential equations with boundary conditions. Solving differential equations with boundary conditions $u_0(t)$,$u_1(t,C1)$, $u_2(t,C2)$, … …..is obtained. Generally the solution of (13) can be determined in the form;

$$u^{\sim(m)} = u_0(t) + \Sigma_{k=1}^{m} u_k(t,C_i) \tag{19}$$

Substituting (19) in (13) we get the following residual:

$$R(t,C_i) = L\left(u^{\sim(m)}(t,C_i)\right) + g(t) + N(u^{\sim(m)}(t,C_i)) \tag{20}$$

If R(t,C_i)=0 then $u^{\sim(m)}(t,C_i)$ is much closed to the exact solution to minimizing the occurred error for nonlinear problem, let;

$$J(C_1, C_2, \qquad ..,C_m) = {}_a^b R^2(t, C_1, C_2, \qquad ,c_m)dt \tag{21}$$

Where a and b are values depending on the given problem. The unknown constants C_i(i = 1,2, … …..,m) can be determined from the conditions:

$$\frac{\partial J}{\partial c_1} = \frac{\partial J}{\partial C_2} = \quad = 0 \tag{22}$$

With these known constants, the approximate solution (of order m) (19) is well determined.

4. Solution using OHAM

Applying (14) into (8),(9) and (10) we get:

$$(1-p)[f''+f'] - H_1(p)[f''' + (f+g)f'' - f'^2 - \frac{y}{a} \ f' + \frac{n}{2} f'' \ -(f''+f')] = 0$$

$$(1-p)[g''+g'] - H_2(p)[(p)[g''' + (f+g)g'' - g'^2 - \frac{y}{a} \ g' + \frac{\eta}{2} g'' - (g''+g') \ ] = 0$$

$$(1-p)[\phi'+\phi] - H_3(p)[\varphi'' + Sc[(f+g)\phi' - \alpha(f'+g')\phi - \frac{y}{a} \ \phi + \frac{\eta}{2}\phi' \ -(\phi'+\phi)] = 0 \tag{23}$$

Where primes denote differentiation with respect to η.

Since the first two equations in (23) are identical, then we take f, g,φ, H_1, H_2 *and* H_3 *as following*:

$$f = f_0 + pf_1 + p^2 f_{2'}$$

$$g = g_0 + pg_1 + p^2 g_{2'}$$

$$\phi = \phi_0 + p\phi_1 + p^2\phi_{2,}$$

$$H_1(p) = pC_1 + p^2 C_{2'}$$

$$H_2(p) = pC_1 + p^2 C_{2'}$$

$$H_3(p) = pC_3 + p^2 C_4 \tag{24}$$

Collecting same powers of p and solving the resulted set of differential equations we obtain;

$$f = 1.5 - e^{-\eta} - \frac{1}{4} c1 \ e^{-2\eta}(2 - 2\,A\,e^{\eta} - 2e^{2\eta} + 2\,Ae^{2\eta} + 4\,e^{\eta}\eta - 2\,A\,e^{\eta}\eta + A\,e^{\eta}\,\eta^2) +$$

$$\frac{1}{96} \ e^{-2\eta}\ (-48\,c1 + 240\,c1^2 + 60\,A\ \ c1^2 - 48\,c2 + 48\,A\,c1\,e^{\eta} - 192\,c1^2\ \ e^{\eta} - 240\,A\,c1^2\ e^{\eta} -$$

$$24\,A^2 c1^2\ e^{\eta} + 48\,A\,c2\,e^{\eta} + 48\,c1\,e^{2\eta} - 48$$

$$A\,c1\,e^{2\eta} + 180\,A\ \ c1^2\ e^{2\eta} + 24\,A^2 c1^2\ e^{2\eta} -$$

$$48\,c2\,e^{2\eta} - 48\,A\,c2\,e^{2\eta} + 72\,A\ \ \eta - \qquad 96\,c1$$

$e^{\eta}\eta + 48\,A\,c1\,e^{\eta}\eta + 288\,c1^2\;e^{\eta}\eta - 192\,A\,c1^2\;e^{\eta}\eta - 24\,A^2c1^2\;e^{\eta}\eta + 96\,c2\;e^{\eta}\eta +$
$48\,A\,c2\,e^{\eta}\eta - 24\,A\,c1\;e^{\eta}\eta^2 - 48\,c1^2\;e^{\eta}\eta^2 = 120\,A\,c1^2\;e^{\eta}\eta^2 - 12\,A^2c1^2\;e^{\eta}\eta^2 -$
$24\,A\,c2\;e^{\eta}\eta^2 - 24\,Ac1^2\;e^{\eta}\eta^3 + 20\,A^2c1^2\;e^{\eta}\eta^3 - 3\;20\,A^2c1^2\;e^{\eta}\eta^4)$

g =

$1.5 - e^{-\eta} - \frac{1}{4}c1\;e^{-2\eta}(2 - 2\,A\,e^{\eta} - 2e^{2\eta} + 2\,Ae^{2\eta} + 4\,e^{\eta}\eta - 2\,A\,e^{\eta}\eta + A\,e^{\eta}\eta^2) +$

$\frac{1}{96}\;e^{-2\eta}(-48\,c1 + 240\,c1^2 + 60\,A\,c1^2 - 48\,c2 + 48\,A\,c1\,e^{\eta} - 192\,c1^2\;e^{\eta} - 240\,A\,c1^2\;e^{\eta} -$

$24\,A^2c1^2\;e^{\eta} + 48\,A\,c2\,e^{\eta} + 48\,c1\;e^{2\eta} - 48$
$A\,c1\;e^{2\eta} + 180\,A\;c1^2\;e^{2\eta} + 24\,A^2c1^2\;e^{2\eta} -$
$48\,c2\,e^{2\eta} - 48\,A\,c2\;e^{2\eta} + 72\,A\eta - \quad 96\,c1$
$e^{\eta}\eta + 48\,A\,c1\,e^{\eta}\eta + 288\,c1^2\;e^{\eta}\eta - 192\,A\,c1^2\;e^{\eta}\eta - 24\,A^2c1^2\;e^{\eta}\eta + 96\,c2\;e^{\eta}\eta +$
$48\,A\,c2\,e^{\eta}\eta - 24\,A\,c1\,e^{\eta}\eta^2 - 48\,c1^2\,e^{\eta}\eta^2 = 120\,A\;c1^2\,e^{\eta}\eta^2 - 12\,A^2\;c1^2\,e^{\eta}\eta^2 -$
$24\,A\,c2\,e^{\eta}\eta^2 - 24\,A\,c1^2\,e^{\eta}\eta^3 + 20\,A^2c1^2\,e^{\eta}\eta^3 - 3\,20\,A^2c1^2\,e^{\eta}\eta^4)$

$\varphi = e^{-\eta} + \frac{1}{4}c3\;e^{-2\eta}(-8\,Sc + 8\,Sc\,e^{\eta} + 8\,Sc\,\alpha e^{\eta} + 4\,e^{\eta}\,\eta - 12\,Sc\,e^{\eta}\,\eta$
$-4\,A\,Sc\,e^{\eta}\eta + A\,Sc\,e^{\eta}\eta^2) + \frac{1}{96}\;e^{-3\eta}(-48\,c1\,c3\,Sc - 192\,c3\,Sc\,e^{\eta}$
$-576\,c1\,c3\,Sc\,e^{\eta} - 96\,A\,c1\,c3\,Sc\,e^{\eta} + 960\,Sc\,c3^2\,e^{\eta} + 192\,c4\,Sc\,e^{\eta}$
$+ 192\,c3\,Sc\,e^{2\eta} + 624\,c1\,c3\,Sc\,e^{2\eta} + 96\,A\,c1\,c3\,sc\,e^{2\eta} - 960\,c3^2\,Sc\,e^{2\eta}$
$- 192\,c4\,Sc\,e^{2\eta} - 384\,c3^2\,Sc^2 - 1152\,c3^2\,Sc^2\,e^{\eta} + 768\,c3^2\,Sc^2\,e^{2\eta}$
$+ 48\,c1\,c3\,Sc\,\alpha + 192\,c3\,Sc\,\alpha e^{\eta} + 576\,c1\,c3\,sc\,\alpha e^{\eta} + 96\,A\,c1\,c3\,sc\,\alpha e^{\eta}$
$- 768\,c3^2 sc\,\alpha e^{\eta} + 192\,c4\,sc\,\alpha e^{\eta} - 192\,c3\,sc\,\alpha e^{2\eta} - 624\,c1\,c3\,sc\,\alpha e^{2\eta}$
$- 96\,A\,c3^2 sc\,\alpha e^{2\eta} + 768\,c3^2\,sc\,\alpha e^{2\eta} - 192\,c4\,sc\,\alpha e^{2\eta} - 576\,c3^2\,Sc^2\,\alpha$
$+ 960\,c3^2\,Sc^2\,\alpha e^{\eta} - 96\,A\,c3^2\,Sc^2\,\alpha e^{\eta} - 384\,c3^2\,Sc^2\,\alpha e^{2\eta}$
$+ 96A\,c3^2\,Sc^2 \propto e^{2\eta} + 192\,c3^2\,Sc^2\,\alpha - 384\,c3^2\,Sc^2\,\alpha^2\,e^{\eta} + 192\,c3^2Sc^2\,\alpha^2\,e^{2\eta}$
$+ 96\,c3\,e^{2\eta}\eta - 288\,c3^2\,e^{2\eta}\eta - 288\,c4\,e^{2\eta}\eta - 384\,c1\,c3\,Sc\,e^{2\eta}\eta$
$- 192\,c3^2\,Sc\,\alpha e^{2\eta}\eta - 288\,c3\,Sc\,e^{2\eta}\eta - 96\,A\,c3\,Sc\,e^{2\eta}\eta - 288\,c1\,c3\,Sc\,e^{2\eta}\eta$
$- 96\,A\,c1\,c3\,Sc\,\alpha e^{2\eta}\eta + 1248\,c3^2\,Sc\,e^{2\eta}\eta + 336\,A\,c3^2\,Sc\,e^{2\eta}\eta + 192c4\,c3\,Sc\,e^{2\eta}\eta$
$- 96\,A\,c4\,Sc\,e^{2\eta}\eta + 576\,A\,c3^2Sc\,e^{2\eta}\eta + 384\,A\,Sc\,c3^2\,Sc\,e^{2\eta}\eta - 960\,c3^2\,Sc\,e^{2\eta}\eta$
$- 384\,A\,c3^2\,Sc\,e^{2\eta}\eta + 384\,c1\,c3\,Sc\,\alpha e^{\eta}\eta + 192\,c3^2\,Sc\,\alpha e^{\eta}\eta$
$+ 288\,c1\,c3\,Sc\,\alpha e^{2\eta}\eta + 96\,A\,c1\,c3\,Sc\,\alpha e^{2\eta}\eta - 192\,c3^2\,Sc\alpha e^{2\eta}\eta$
$+ 576\,c3^2Sc^2\,\alpha e^{\eta}\eta - 288\,A\,c3^2\,Sc^2\,\alpha e^{2\eta}\eta + 384\,c3^2\,Sc^2\,\alpha e^{2\eta}\eta$
$+ 192\,A\,c3^2Sc^2\,\alpha e^{\eta}\eta + 48\,c3^2\,e^{2\eta}\eta^2 + 48\,A\,c1\,c3\,Sc\,e^{\eta}\eta^2$
$+ 24\,A\,c3\,Sc\,e^{2\eta}\eta^2 - 240\,Sc\,c3^2\,e^{2\eta}\eta^2 - 192\,A\,Sc\,c3^2\,e^{2\eta}\eta^2$
$- 24\,A\,c4\,Sc\,e^{2\eta}\eta^2 - 48\,A\,c3^2\,Sc^2\,e^2\eta^2 + 288\,c3^2\,Sc^2\,e^{2\eta}\eta^2$
$+ 408\,A\,c3^2\,Sc^2\,e^{2\eta}\eta^2 + 72\,A^2\,c3^2\,Sc^2\,e^{2\eta}\eta^2 - 48A\,c1\,c3\,Sc\,\alpha e^{\eta}\eta^2$
$+ 48\,A\,c3^2\,Sc^2\,e^{2\eta}\eta^2 - 48\,A\,c3^2\,Sc^2\,\alpha e^{2\eta}\eta^2 + 24\,A\,Sc\,c3^2\,e^{2\eta}\eta^3$
$- 64\,A\,c3^2\,Sc^2\,e^{2\eta}\eta^3 - 32\,A^2\,c3^2\,Sc^2\,e^{2\eta}\eta^3 + 3\,A^2\,c3^2\,Sc^2\,e^{2\eta}\eta^4$

Where $A = \frac{\gamma}{a}$

Computations have been carried out for various values of the dynamic parameter (A), the concentration parameter (α), and the schmidt number (Sc).

Results for –f’’(0) or –g’’(0) are computed for various values of the dynamic parameter (A) in Table(1).

Table 1: Values of –f ''(0) for a various values of A

A	-f ''(0)
0	1.000000
1	0.977727
2	0.415989

Also computations of the concentration surface gradient for different values of the Schmidt number and the concentration parameter (α) are shown in tables (2-4).

Table 2: Values of - φ'(0) for various values of (α) at Sc=0.6,1.8,6 and A=0

Sc	α	-φ'(0)
0.6	-2	2.1342
0.6	-1	-0.45032
0.6	0	1.04137
0.6	1	1.74844
0.6	2	2.28211
1.8	-2	1.64041
1.8	-1	2.03696
1.8	0	0.998902
1.8	1	2.48314
1.8	2	4.0995
6	-2	3.09714
6	-1	2.68617
6	0	2.15111
6	1	-1.85212
6	2	1.44507

Table 3: Values of - φ'(0) for various values of (α) at Sc=0.6, 1.8,6 and A=1

Sc	α	-φ'(0)
0.6	-2	0.923513
0.6	-1	0.995101
0.6	0	1.738
0.6	1	2.15747
0.6	2	2.41578
1.8	-2	1.36891
1.8	-1	0.500505
1.8	0	1.83653
1.8	1	2.61294
1.8	2	2.89841
6	-2	2.55492
6	-1	1.43083
6	0	1.36863
6	1	1.59819
6	2	2.65201

Table 4: Values of -φ'(0) for various values of (α) at Sc=0.6,1.8,6 and A=2

Sc	α	-φ'(0)
0.6	-2	2.65201
0.6	-1	1.72106
0.6	0	2.28143
0.6	1	2.53194
0.6	2	2.95135
1.8	-2	1.06287

Table 4: Values of -φ'(0) for various values of (α) at Sc=0.6,1.8,6 and A=2 - continued

1.8	-1	0.569166
1.8	0	1.7107
1.8	1	1.30743
1.8	2	5.40516
6	-2	2.11668
6	-1	0.618507
6	0	-11.1735
6	1	7.24249
6	2	2.79353

4. Discussions

The influence of the dynamic parameter, the concentration parameter (α), and the Schmidt number (Sc) on the dimensionless concentration is shown in figs. (2-10).

Figure (2) shows the effect of the concentration parameter (α) on the dimensionless concentration when the dynamic parameter (A) =1, the Schmidt number (Sc) = 0.6. We observe that the decrease of the concentration parameter decreases the concentration for any case of the concentration parameter.

Figure 2: A=1, Sc=0.6,α=0,-1,-2

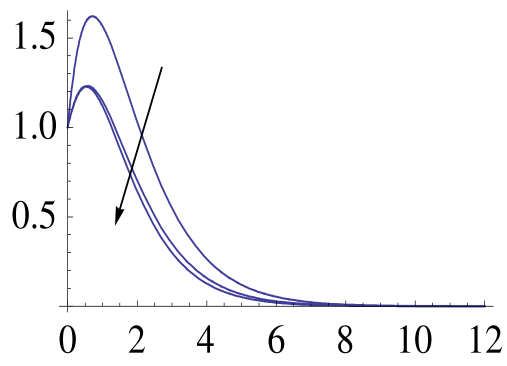


Figure (3) shows the effect of the concentration parameter (α) on the dimensionless concentration when the dynamic parameter (A) =1, the schmidt number (Sc) = 1.8.We observe that the increase of the concentration parameter decreases the concentration for any case of the concentration parameter.

Figure 3: A=1, Sc=1.8,α=-2,-1,1

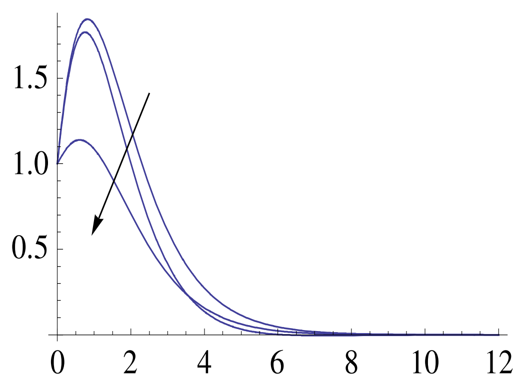


Figure (4) shows the effect of the concentration parameter (α) on the dimensionless concentration when the dynamic parameter (A) =1, the scmidt number (Sc) = 6.We observe that the increase of the concentration parameter decreases the concentration for the case of positive values of the concentration parameter.

Figure 4: A=1, Sc=6, α=0, 1, 2

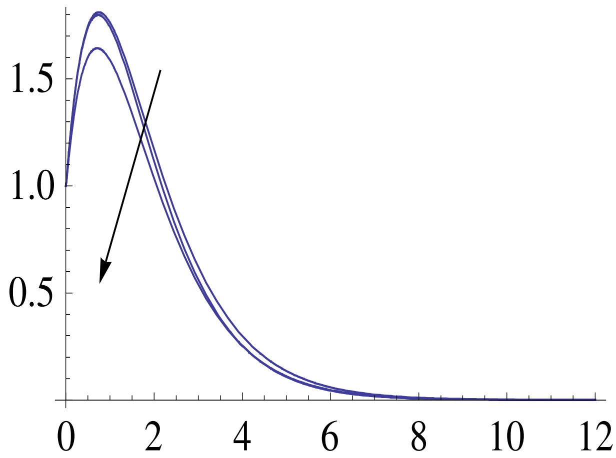


Figure (5) shows the effect of the concentration parameter (α) on the dimensionless concentration when the dynamic parameter (A) = 0 (steady case), the scmidt number (Sc) = 0.6.We observe that the decrease of the concentration parameter decreases the concentration for any case of the concentration parameter.

Figure 5: A=0, Sc=0.6, α=1,0,-1

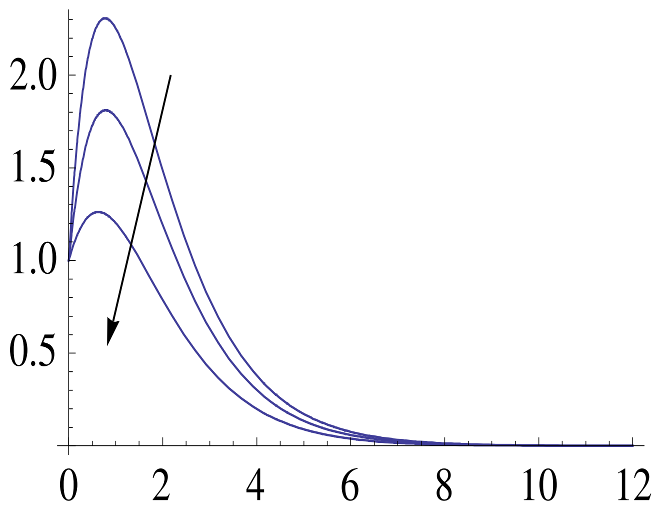


Figure (6) shows the effect of the concentration parameter (α) on the dimensionless concentration when the dynamic parameter (A) = 0 (steady case), the Scmidt number (Sc) = 1.8.We observe that the decrease of the concentration parameter decreases the concentration for the for values of $\eta > 2$.

Figure 6: A=0, Sc=1.8, α=0,-1, 1

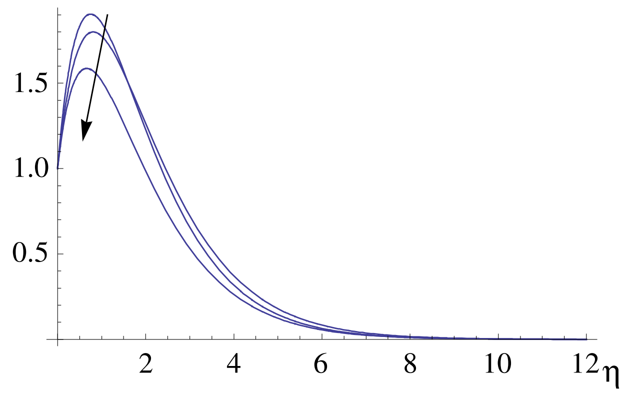


Figure (7) shows the effect of the concentration parameter (α) on the dimensionless concentration when the dynamic parameter (A) =0 (steady case), the Scmidt number (Sc) = 6.We observe that the decrease of the concentration parameter decreases the concentration for the negative values of the concentration parameter.

Figure 7: A=0, Sc=6, α=0,-1, 2

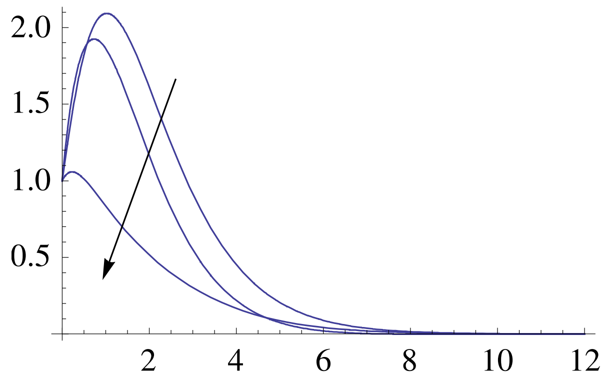


Figure (8) shows the effect of the concentration parameter (α) on the dimensionless concentration when the dynamic parameter (A) =2, the scmidt number (Sc) = 0.6. We observe that the decrease of the concentration parameter decreases the concentration for any case of the concentration parameter.

Figure 8: A=2, Sc=0.6, α=0,-1,-2

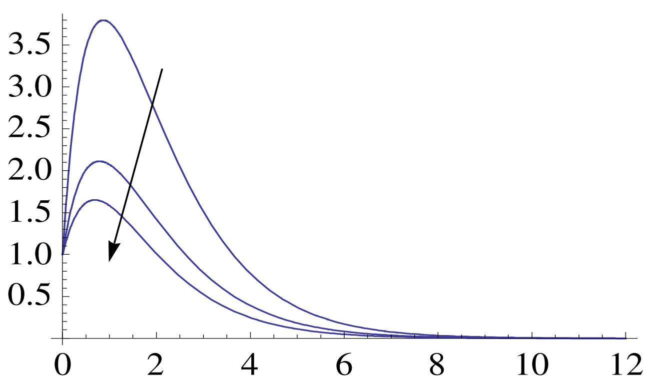


Figure (9) shows the effect of the concentration parameter (α) on the dimensionless concentration when the dynamic parameter (A) =2, the scmidt number (Sc) =1.8.We observe that the increase of the concentration parameter decreases the concentration for the negative values of the concentration parameter at values of η<2.1..

Figure 9: A=2, Sc=1.8,α=-2,0,2

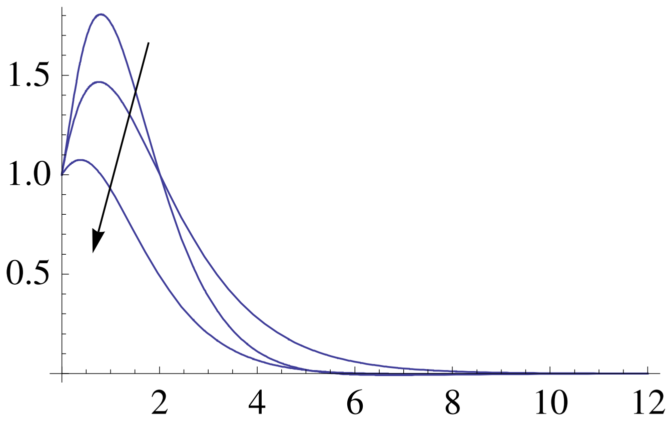
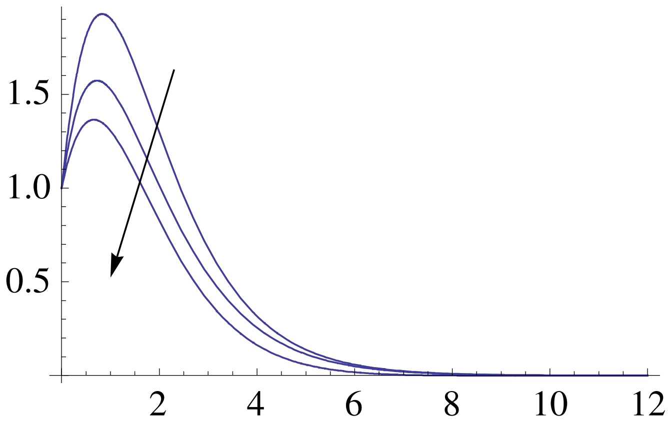


Figure (10) shows the effect of the concentration parameter (α) on the dimensionless concentration when the dynamic parameter (A) =2, the scmidt number (Sc) = 6.We observe that the increase of the concentration parameter decreases the concentration for any case of the concentration parameter.

Figure 10: A=2,Sc=6,α=-2,1,2



Discussions

The influence of the dynamic parameter (A), the concentration parameter (α) and the schmidt number (Sc) on the dimensionless velocity and the dimensionless concentration are shown in figures (2-10).

Figures (2-4) show the effect of the concentration parameter (α), on the concentration for value of the dynamic parameter (A) =1. We observe that the decrease of the negative values of the concentration parameter (α) or the increase of the positive values of the concentration parameter (α) will decrease the concentration.

Figures (5-7) show the effect of the concentration parameter (α), on the concentration for value of the dynamic parameter (A) =0 (the steady case). We observe that the decrease of the negative values of the concentration parameter (α) decreases the concentration.

Figure (8) show the effect of the concentration parameter (α), on the concentration for value of the dynamic parameter (A) = 2 and schmidt number (Sc) = 0.6. We observe that the decrease of the values of the concentration parameter (α) decreases the concentration.

Figure (9) shows the effect of the concentration parameter (α), on the concentration for value of the dynamic parameter (A) = 2 and schmidt number (Sc) = 1.8. We observe that the decrease of the values of the concentration parameter (α) decreases the concentration at values of $\eta<2.1$.

Figure (10) shows the effect of the concentration parameter (α), on the concentration for value of the dynamic parameter (A) = 2 and schmidt number (Sc) = 6.We observe that the increase of the values of the concentration parameter (α) decreases the concentration.

In general the decrease of the negative values of the concentration parameter (α) decreases the concentration. There exists a certain value of (η) at which we get the maximum value of the concentration; this value depends on the value of the Schmidt number. The value of the concentration tends to zero as the dimensionless parameter (η) tends to infinity (∞).

Table (1) shows that the values of $-f''(0)$ and $-g''(0)$ decreases with the increase of the dynamic parameter (A)

Tables (2-4) show that the values of concentration gradient at the surface $-\varphi'(0)$ decreases with the increase of the negative values of the concentration paramater(α) and increases with the increase of the positive values of the concentration parameter (α)

Conclusion

Optimal Homotopy Analysis Method has been applied to study the effects of the dynamic parameter (A), the concentration parameter (α) and the Schmidt number (Sc) on the velocity and the concentration. It is found that there are considerable effects for these parameters on the velocity and concentration.

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